SIGNALS, SYSTEMS, AND TRANSFORMS

FOURTH EDITION
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To

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The basic structure and philosophy of the previous editions of *Signals, Systems, and Transforms* are retained in the fourth edition. New examples have been added and some examples have been revised to demonstrate key concepts more clearly. The wording of passages throughout the text has been revised to ease reading and improve clarity. In particular, we have revised the development of convolution and the Discrete Fourier Transform. Biographical information about selected pioneers in the fields of signal and system analysis has been added in the appropriate chapters. References have been removed from the end of each chapter and are collected in Appendix I.

Many end-of-chapter problems have been revised and numerous new problems are provided. Several of these new problems illustrate real-world concepts in digital communications, filtering, and control theory. The end-of-chapter problems have been organized so that multiple similar problems are provided. The answer to at least one of each set of similar problems is provided in Appendix H. The intent is to allow students to develop confidence by gaining immediate feedback about their understanding of new material and concepts. All MATLAB examples have been updated to ensure compatibility with the Student Version Release 14.

A companion web site at [http://www.ee.washington.edu/class/SST_textbook/textbook.html](http://www.ee.washington.edu/class/SST_textbook/textbook.html) contains sample laboratories, lecture notes for Chapters 1–7 and Chapters 9–12, and the MATLAB files listed in the textbook as well as several additional MATLAB files. It also contains a link to a second web site at [http://www.ee.washington.edu/class/235dl/](http://www.ee.washington.edu/class/235dl/), which contains interactive versions of the lecture notes for Chapters 1–7. Here, students and professors can find worked-out solutions to all the examples in the lecture notes, as well as animated demonstrations of various concepts including transformations of continuous-time signals, properties of continuous-time systems (including numerous examples on time-invariance), convolution, sampling, and aliasing. Additional examples for discrete-time material will be added as they are developed.

This book is intended to be used primarily as a text for junior-level students in engineering curricula and for self-study by practicing engineers. It is assumed that
the reader has had some introduction to signal models, system models, and differential equations (as in, for example, circuits courses and courses in mathematics), and some laboratory work with physical systems.

The authors have attempted to consistently differentiate between signal and system models and physical signals and systems. Although a true understanding of this difference can be acquired only through experience, readers should understand that there are usually significant differences in performance between physical systems and their mathematical models.

We have attempted to relate the mathematical results to physical systems that are familiar to the readers (for example, the simple pendulum) or physical systems that students can visualize (for example, a picture in a picture for television). The descriptions of these physical systems, given in Chapter 1, are not complete in any sense of the word; these systems are introduced simply to illustrate practical applications of the mathematical procedures presented.

Generally, practicing engineers must, in some manner, validate their work. To introduce the topic of validation, the results of examples are verified, using different procedures, where practical. Many homework problems require verification of the results. Hence, students become familiar with the process of validating their own work.

The software tool MATLAB is integrated into the text in two ways. First, in appropriate examples, MATLAB programs are provided that will verify the computations. Then, in appropriate homework problems, the student is asked to verify the calculations using MATLAB. This verification should not be difficult because MATLAB programs given in examples similar to the problems are applicable. Hence, another procedure for verification is given. The MATLAB programs given in the examples may be downloaded from http://www.ee.washington.edu/class/SST_textbook/textbook.html. Students can alter data statements in these programs to apply them to the end-of-chapter problems. This should minimize programming errors. Hence, another procedure for verification is given. However, all references to MATLAB may be omitted, if the instructor or reader so desires.

Laplace transforms are covered in Chapter 7 and $z$-transforms are covered in Chapter 11. At many universities, one or both transforms are introduced prior to the signals and systems courses. Chapters 7 and 11 are written such that the material can be covered anywhere in the signals and systems course, or they can be omitted entirely, except for required references.

The more advanced material has been placed toward the end of the chapters wherever possible. Hence, this material may be omitted if desired. For example, Sections 3.7, 3.8, 4.6, 5.5, 7.9, 10.7, 12.6, 12.7, and 12.8 could be omitted by instructors without loss of continuity in teaching. Further, Chapters 8 and 13 can be skipped if a professor does not wish to cover state-space material at the undergraduate level.

The material of this book is organized into two principal areas: continuous-time signals and systems, and discrete-time signals and systems. Some professors prefer to cover first one of these topics, followed by the second. Other professors prefer to cover continuous-time material and discrete-time material simultaneously.
The authors have taken the first approach, with the continuous-time material covered in Chapters 2–8, and the discrete-time material covered in Chapters 9–13. The material on discrete-time concepts is essentially independent of the material on continuous-time concepts so that a professor or reader who desires to study the discrete-time material first could cover Chapters 9–11 and 13 before Chapters 2–8. The material may also be arranged such that basic continuous-time material and discrete-time material are intermixed. For example, Chapters 2 and 9 may be covered simultaneously and Chapters 3 and 10 may also be covered simultaneously.

In Chapter 1, we present a brief introduction to signals and systems, followed by short descriptions of several physical continuous-time and discrete-time systems. In addition, some of the signals that appear in these systems are described. Then a very brief introduction to MATLAB is given.

In Chapter 2, we present general material basic to continuous-time signals and systems; the same material for discrete-time signals and systems is presented in Chapter 9. However, as stated above, Chapter 9 can be covered before Chapter 2 or simultaneously with Chapter 2. Chapter 3 extends this basic material to continuous-time linear time-invariant systems, while Chapter 10 does the same for discrete-time linear time-invariant systems.

Presented in Chapters 4, 5, and 6 are the Fourier series and the Fourier transform for continuous-time signals and systems. The Laplace transform is then developed in Chapter 7. State variables for continuous-time systems are covered in Chapter 8; this development utilizes the Laplace transform.

The z-transform is developed in Chapter 11, with the discrete-time Fourier transform and the discrete Fourier transform presented in Chapter 12. However, Chapter 12 may be covered prior to Chapter 11. The development of the discrete-time Fourier transform and discrete Fourier transform in Chapter 12 assumes that the reader is familiar with the Fourier transform. State variables for discrete-time systems are given in Chapter 13. This material is independent of the state variables for continuous-time systems of Chapter 8.

In Appendix A, we give some useful integrals and trigonometric identities. In general, the table of integrals is used in the book, rather than taking the longer approach of integration by parts. Leibnitz’s rule for the differentiation of an integral and L’Hôpital’s rule for indeterminate forms are given in Appendix B and are referenced in the text where needed. Appendix C covers the closed forms for certain geometric series; this material is useful in discrete-time signals and systems. In Appendix D, we review complex numbers and introduce Euler’s relation, in Appendix E the solution of linear differential equations with constant coefficients, and in Appendix F partial-fraction expansions. Matrices are reviewed in Appendix G; this appendix is required for the state-variable coverage of Chapters 8 and 13. As each matrix operation is defined, MATLAB statements that perform the operation are given. Appendix H provides solutions to selected chapter problems so that students can check their work independently. Appendix I lists the references for the entire text, arranged by chapter.

This book may be covered in its entirety in two 3-semester-hour courses, or in quarter courses of approximately the equivalent of 6 semester hours. With the omission of appropriate material, the remaining parts of the book may be covered with